

## Inhaltsverzeichnis

Standard IO system.....	2
Complex Numbers.....	2
Adding Matrix Powers.....	2
Square and Multiply.....	2
Backtracking.....	2
Number of Digits.....	3
Convex Hull (Graham Scan 1).....	3
Convex Hull (Graham Scan 2).....	4
Binomial Sums.....	4
Matrix Class.....	4
Fast prime generation.....	5
Special Sums.....	6

**Standard IO system**

```
import java.util.*;
```

```
class Main
```

```
{
    public static void main(String... strings)
    {
        Scanner sc = new Scanner(System.in);

        int n = sc.nextInt();

        System.out.println(String.format("%d", n));
    }
}
```

---

**Complex Numbers**
$$z = r (\cos(\phi) + i \sin(\phi)) = r e^{i \phi} = (a + b i)$$
$$r = \sqrt{a^2 + b^2}$$
$$\phi = \pm \arctan2(b / a)$$

---

**Adding Matrix Powers**
$$A^1 + A^2 + \dots + A^n = S[n]$$
$$S[n] = S[n/2] + A^{(n/2)} * S[n/2]; \text{ // when } n \text{ is even}$$
$$S[n] = A + A * S[n-1]; \text{ // when } n \text{ is odd}$$

---

**Square and Multiply**

// computes a number to the power of e.

```
public Number pow(int e)
{
    Number result = this;
    List<Number> tomult = new ArrayList<Number>();
    while (e > 1)
    {
        if ((e & 1) == 0)
        {
            result = result.mult(result);
            e >>= 1;
        }
        else
        {
            tomult.add(new Number(this.clone()));
            e--;
        }
    }
    for (Number c : tomult)
        result = result.mult(c);
    return result;
}
```

---

**Backtracking**

```

// start with track(new ArrayList<Integer>(), 0, 0);
// it will fill „solutions“ with all column-solutions sorted by row.
private static List<List<Integer>> solutions;
private static void track(List<Integer> mySolution, int row, int max)
{
    if(row >= max)
    {
        solutions.add(mySolution);
        return;
    }

    for(int col=0; col<max; col++)
    {
        if(possibleSolution(mySolution, row, col))
        {
            List<Integer> s = new ArrayList<Integer>(mySolution);
            s.add(col);
            track(s, row + 1, max);
        }
    }
}
// you can give the criteria for a possible solution here
private static void possibleSolution(List<Integer> mySolution, int row, int col)
{
    if( „adding an element to (row,col) is possible in mySolution“ )
        return true;
    else
        return false;
}

```

### Number of Digits

Number of Digits(N) = Math.floor( Math.log10( N ) ) + 1

### Point Class for Graham Scan

```

public static Point firstPoint;
// Point class representing one point in a cloud of points. Useful for graham scan.
public static class Point implements Comparable<Point>
{
    public int y;
    public int x;

    public Point(int x, int y)
    {
        this.x = x;
        this.y = y;
    }

    // determines the polar angle between this and a predefined point.
    public double angle()
    {
        return Math.atan2((double) (y - firstPoint.y), (double) (x - firstPoint.x));
    }

    public double size()
    {
        return Math.sqrt((long) (x - firstPoint.x) * (long) (x - firstPoint.x) + (long) (y -
firstPoint.y) * (long) (y - firstPoint.y));
    }
}

```

```

@Override
public int compareTo(Point p2)
{
    if (Math.abs(this.cotangent() - p2.cotangent()) < 1e-5)
        if (this.size() < p2.size())
            return -1;
        else if (this.size() > p2.size())
            return 1;
        else
            return 0;
    if (this.cotangent() < p2.cotangent())
        return -1;
    return 1;
}
}

```

### Convex Hull (Graham Scan)

```

List<Point> pointList = new ArrayList<Point>();
// start testcase, read in all points and add them to pointList.
// also determine the point with the lowest y coordinate (and most left x if y is the same).
...
// move the point with lowest y coordinate to index 0
Collections.swap(pointList, 0, pointList.indexOf(pWithLowestY));

// save the first point for size and angle measurement
firstPoint = pWithLowestY;

// sort points by polar coordinate angle, see „point class for graham scan“
Collections.sort(pointList);

// add the first point at the end to close the polygon (not absolutely necessary)
pointList.add(new Point(pWithLowestY.x, pWithLowestY.y));

// start with graham scan, always take the most right point from the current point.
int i = 2;
while (i < pointList.size())
{
    // if the points are in one line, remove the middle point
    if (ccw(pointList.get(i - 2), pointList.get(i - 1), pointList.get(i)) == 0)
    {
        pointList.remove(i - 1);
    }
    else
    {
        // if the points are directed to the left, its ok
        if (ccw(pointList.get(i - 2), pointList.get(i - 1), pointList.get(i)) > 0)
        {
            i++;
        }
        // if they are directed "concave", remove the middle one and go back
        else
        {
            pointList.remove(i - 1);
            i--;
        }
    }
}

// now the pointList contains only points on the hull
for (Point p : pointList)
    System.out.println(String.format("%d %d", p.x, p.y));

```

---

### Binomial Sums

$$\sum_{k=0}^n \binom{n}{k} = 2^n$$

$$\sum_{k=1}^n k \binom{n}{k} = n 2^{n-1}$$

$$(x+1)^n = \sum_{i=0}^n \binom{n}{i} x^i$$

### Matrix Class

// Matrix class. Can add and multiply other matrices. For computing  $A^n$ , see „Square and Multiply“.

```
public class Matrix
{
    private final int[][] elements;
    private final int rows,cols;
    public Matrix(int n, int m)
    {
        elements = new int[n][m];
        this.rows = n;
        this.cols = m;
    }

    // multiplies a matrix with the current and returns the result
    public Matrix mult(Matrix matrix)
    {
        Matrix result = new Matrix(cols, matrix.rows);

        for(int lrows=0; lrows < rows; lrows++)
        {
            for(int lcols=0; lcols < cols; lcols++)
            {
                result.elements[lrows][lcols] = 0;
                for(int k=0;k<cols; k++)
                    result.elements[lrows][lcols] += elements[lrows][k]*matrix.elements[k]
[lcols];
            }
        }
        return result;
    }

    // adds a matrix to the current and returns the result
    public Matrix add(Matrix matrix)
    {
        Matrix result = new Matrix(cols, rows);
        for(int lrows=0; lrows<rows; lrows++)
            for(int lcols=0; lcols<cols; lcols++)
                result.elements[lrows][lcols] = this.elements[lrows][lcols] + matrix.elements[lrows]
[lcols];
        return result;
    }
}
```

### Fast prime generation

```
// Calculate all prime numbers in the range 1 to MAX_PRIME
// using the sieve of Eratosthenes.
// numbers in the "numbers" array that are not prime are set to -1.
static final int MAX_PRIME = 5000000;
static int[] numbers = new int[MAX_PRIME+5];

private static void generatePrimes()
{
    numbers[0] = -1;
    numbers[1] = -1;
    for (int i = 2; i <= MAX_PRIME+1; i++)
    {
        if (numbers[i] >= 0)
        {
            numbers[i] = i;
            for (int k = 2; k * i <= MAX_PRIME+1; k++)
            {
                numbers[k * i] = -1;
            }
        }
    }
}
```

---

### Special Sums

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{\infty} \frac{x^i}{i} = -\ln(1-x)$$